Math 546, Exam 2, Summer, 2001

PRINT Your Name:______________________________

There are 10 problems on 5 pages. Each problem is worth 5 points.

1. Define "cyclic group". Use complete sentences.

2. Define "center". Use complete sentences.

3. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If every proper subgroup H of a group G is abelian, then G is abelian.

4. State Lagrange's Theorem.

5. True or False (If true, then prove it. If false, then give a counterexample.) If H and K are non-zero subgroups of $(\mathbb{R},+)$, then the intersection of H and K is non-zero.

6. Find all of the subgroups of $U_9 = \{z \in \mathbb{C} \mid z^9 = 1\}$. Explain why you are certain that you have found all of the subgroups.

7. Let H be a subgroup of the group G. Let a be a fixed element of G and let
$$K = \{aha^{-1} \mid h \in H\}.$$
Prove that K is a subgroup of G.

8. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If H and K are subgroups of a group G, then the intersection $H \cap K$ is also a subgroup of G.

9. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If H and K are subgroups of a group G, then the union $H \cup K$ is also a subgroup of G.

10. Let $W = \mathbb{R} \setminus \{2\}$. Define $*$ on W by $a * b = ab - 2a - 2b + 6$. Prove that $(W, *)$ is a group.