Math 546, Exam 1, Summer, 2001

PRINT Your Name:________________________________

There are 9 problems on 6 pages. Problems 1–5 are worth 6 points each. Each of the other problems is worth 5 points.

1. Define “group”. Use complete sentences.

2. Define “subgroup”. Use complete sentences.

3. Let G be the subgroup $\{1,-1,i,-i\}$ of the group of non-zero complex numbers under multiplication.
   (a) Record the multiplication table for G.
   (b) In class we found 8 subgroups of the group D_4. Three of these subgroups had four elements, just like the group G. Does the multiplication table of G look more like the multiplication table of $H = \{\text{id}, \rho, \rho^2, \rho^3\}$ or more the multiplication table of $K = \{\text{id}, \sigma\rho, \rho^2, \sigma\rho^3\}$. Explain your answer. (I do not need to see a large number of details.)

4. Let $T = \mathbb{R} \setminus \{1\}$. Define $*$ on T by $a * b = ab - a - b + 2$. Proof that $(T, *)$ is a group.

5. Recall that D_3 is the smallest subgroup of the group of rigid motions which contains ρ and σ, where ρ is rotation counter clockwise by $120°$ fixing the origin and σ is reflection of the xy plane across the x axis. List 4 subgroups of D_3 in addition to D_3 and $\{\text{id}\}$. (I do not need to see any details.)

6. Let x and y be elements of the group $(G, *)$. Suppose that the inverse of x is called x^{-1} and the inverse of y is called y^{-1}. Write the inverse of $x * y$ in terms of x^{-1} and y^{-1}. Explain why your answer is correct.

7. Define $*$ on $\mathbb{Z} \setminus \{0\}$ by $a * b = ab$. Is $(\mathbb{Z}, *)$ a group? Why or why not?

8. Let $\mathbb{R}^{\text{pos}}$ be the set of positive real numbers. Define $*$ on $\mathbb{R}^{\text{pos}}$ by $a * b = ab$. Is $(\mathbb{R}^{\text{pos}}, *)$ a group? Why or why not?

9. Let $\mathbb{R}^{\text{pos}}$ be the set of positive real numbers. Define $*$ on $\mathbb{R}^{\text{pos}}$ by $a * b = a/b$. Is $(\mathbb{R}^{\text{pos}}, *)$ a group? Why or why not?