1. Write \((1, 4)(1, 2, 3, 4, 5)(4, 6, 7)\) as a product of disjoint cycles.

2. Prove that the group of real numbers under addition is isomorphic to the group of positive real numbers under multiplication.

Problems 3, 4, and 5 all refer to the following situation: Let \(S\) be a set and let \(B\) be a subset of \(S\). Define
\[
H = \{ \sigma \in \text{Sym}(S) \mid \sigma(b) \in B \text{ for all } b \in B \}.
\]

3. Suppose \(S = \{1, 2, 3, 4, 5\}\) and \(B = \{2, 3\}\). LIST the elements of \(H\).

4. Return to the general situation as described before problem three. Assume that the set \(S\) is finite. Prove that \(H\) is a subgroup of \(\text{Sym}(S)\).

5. Return to the general situation as described before problem three. Assume that the set \(S\) is infinite. Give an example in which \(H\) is NOT a subgroup of \(\text{Sym}(S)\). Explain your example thoroughly.

6. Let \(G\) be a group and \(a\) be a fixed element of \(G\). Define \(\phi: G \rightarrow G\) by \(\phi(g) = aga^{-1}\) for all \(g \in G\). Prove that \(\phi\) is a group isomorphism.

7. Give two non-isomorphic groups of order 36. Explain why the groups are not isomorphic.

8. List the elements of the group \(S_3 \times \mathbb{Z}_2\). What is the order of each element?

9. Exhibit an isomorphism \(\phi: U \rightarrow G\), where \(U\) is the unit circle group and \(G\) is a subgroup of \(\text{GL}_2(\mathbb{R})\). Tell me what \(G\) is. Tell me what \(\phi\) is. Prove that \(\phi\) is an isomorphism.

10. Exhibit an isomorphism \(\phi: (\mathbb{R} \setminus \{0\}, \times) \rightarrow (\mathbb{R} \setminus \{-2\}, \ast)\), where \(a \ast b = ab + 2a + 2b + 2\). Tell me what \(\phi\) is and prove that \(\phi\) is an isomorphism.