1. (5 points) Define “order”. Use complete sentences.

2. (5 points) List ALL of the generators of \((\mathbb{Z}_8, +)\). No explanation is needed.

3. (5 points) List ALL of the subgroups of \((U_{12}, \times)\). No explanation is needed.

4. (5 points) Is \((\mathbb{Z}_{15}, \times)\) a cyclic group? Explain.

5. (5 points) Recall that each element of \(\mathbb{C}\) is a point on the complex plane. Give a geometric description of the left cosets of \(U\) in \((\mathbb{C} \setminus \{0\}, \times)\).

6. (5 points) PROVE that every subgroup of \((\mathbb{Z}, +)\) is cyclic.

7. (4 points) Let \(m\) and \(n\) be positive integers and let \(d\) be the greatest common divisor of \(m\) and \(n\). PROVE that there exist integers \(r\) and \(s\) with \(d = rm + sn\).

8. Let \(a\) and \(b\) be elements of finite order in the group \(G\).
   (a) (4 points) List two hypotheses (Hypothesis (1) and Hypothesis (2)) with the property that if Hypothesis (1) and Hypothesis (2) both hold, then the order of \(ab\) is equal to the order of \(a\) times the order of \(b\).
   (b) (4 points) Give an example where Hypothesis (1) holds, Hypothesis (2) fails to hold, and the conclusion also fails to hold.
   (c) (4 points) Give an example where Hypothesis (2) holds, Hypothesis (1) fails to hold, and the conclusion also fails to hold.
   (d) (4 points) Prove the result which you stated in (a).