1. STATE and PROVE Lagrange’s Theorem.

2. Let $G$ be a group and $g$ be an element of $G$.
   (a) Define the center, $Z(G)$, of $G$.
   (b) Define the centralizer, $C_G(g)$, of $g$ in $G$.
   (c) Is it always true that $C_G(g) \subseteq Z(G)$? If yes, prove it. If no, give a counterexample.
   (d) Is it always true that $Z(G) \subseteq C_G(g)$? If yes, prove it. If no, give a counterexample.

3. (Yes or No. If yes, PROVE it. If no, give a COUNTEREXAMPLE.) Let $H$ and $K$ be subgroups of the group $G$ with $H \neq \{\text{id}\}$ and $K \neq \{\text{id}\}$. Is it always true that $H \cap K \neq \{\text{id}\}$?

4. (Yes or No. If yes, PROVE it. If no, give a COUNTEREXAMPLE.) Let $G$ be a group in which every proper subgroup is cyclic. Does the group $G$ have to be cyclic?

5. (Yes or No. If yes, PROVE it. If no, give a COUNTEREXAMPLE.) Let $G$ be a group and let $S$ be the subset $S = \{x \in G \mid x^2 = \text{id}\}$ of $G$. Is $S$ always a subgroup of $G$?

6. (Yes or No. If yes, PROVE it. If no, give a COUNTEREXAMPLE.) Let $G$ be an abelian group and let $S$ be the subset $S = \{x \in G \mid x^2 = \text{id}\}$ of $G$. Is $S$ always a subgroup of $G$?

7. List the left cosets of the subgroup $H = \{\text{id}, \rho, \rho^2, \rho^3\}$ in the group $G = D_4$. I do not need to see many details.