Math 546, Exam 1, Spring, 2004

PRINT Your Name:________________________________

There are 8 problems on 5 pages. Problems 1 and 2 are worth 7 points each. Each of the other problems is worth 6 points.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your exam outside my office tomorrow by about noon, you may pick it up any time between then and the next class. **Let me know if you are interested.**

I will post the solutions on my website at about 6:30 PM today.

1. Define "group". Use complete sentences.

2. Exhibit a group G and two elements a and b of G with $(ab)^2 \neq a^2b^2$.

3. Let G be a group and let H and K be subgroups of G . Is the intersection $H \cap K$ always a subgroup of G ? If yes, prove the result. If no, show a counterexample.

4. Let G be a group and let H and K be subgroups of G . Is the union $H \cup K$ always a subgroup of G ? If yes, prove the result. If no, show a counterexample.

5. Let $S = \mathbb{R} \setminus \{-2\}$. Define $*$ on S by $a * b = ab + 2a + 2b + 2$. Prove that $(S, *)$ is a group.

6. Define "centralizer". Use complete sentences.

7. Let $G = \{\left[\begin{smallmatrix} a & c \\ 0 & b \end{smallmatrix}\right] \mid a, b, c \in \mathbb{R} \text{ with } a \neq 0 \text{ and } b \neq 0\}$. The set G forms a group under matrix multiplication. (You do not have to prove this.) Find the centralizer of $g = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ in G .

8. Recall that D_3 is the smallest subgroup of the group of rigid motions which contains ρ and σ , where ρ is rotation counter clockwise by $120°$ fixing the origin and σ is reflection of the xy plane across the x axis. Recall also that the elements of D_3 are: id , ρ , ρ^2 , σ , $\sigma\rho$, and $\sigma\rho^2$. Let H be the following subset of D_3 :
$$H = \{g^3 \mid g \in D_3\}.$$
   (a) List the elements of H .
   (b) Is H a subgroup of D_3 ? Explain.