12. What is the order of the element \((\cos \frac{\pi}{3} + i\sin \frac{\pi}{3}, \rho)\) in the group \(U_6 \times D_4\)?
Explain your answer.

\[
U \text{ has order } 6, \text{ and } \rho \text{ has order } 4. \quad (U, \rho)^n = ((U^n, \rho^n), \text{ if } (U, \rho)^n \neq (1, 1)) \implies n \text{ must divide } 6 \text{ and } 4 \text{ must divide } \rho. \text{ The smallest such } n \text{ is } n = 12.
\]

13. Let \(G\) be the group \(Z_4 \times Z_{10}\). Let \(N\) be the subgroup \(<(2, 2)>\) of \(G\). What is the order of the element \((1, 2) + N\) in the group \(G/N\)? Explain your answer.

\[
N = \{(0,0), (2,0), (2,2), (4,0), (4,2), (6,0), (6,2), (8,0), (8,2)\}
\]

\[
(1, 2) + N \text{ is not the identity element of } G,
\]

\[
(1, 2) + N + (1, 2) + N = (2, 4) + N = (0, 0) + N \text{ is the identity element of } G
\]

So \((1, 2) + N\) has order \(2\) in \(G/N\)

14. Let \((\mathbb{R}^+, \times)\) represent the group of positive real numbers under multiplication. Does \((\mathbb{R}^+, \times)\) contain any non-cyclic subgroups? If not, explain why not. If so, exhibit such a subgroup and explain why the subgroup is not cyclic.

The group \(\mathbb{R}^+\) is not cyclic. Cyclic groups are countable or finite.

A different argument is: Suppose \(g\) generates \(\mathbb{R}^+\). Well \(\mathbb{R} = \mathbb{R}^+\)

so \(\sqrt{g} = gh\) for some integer \(n\)

so \(g = g^{2n}\) so \(g(g^{2n-1} - 1) = 0\)

But \(g > 0\) so \(g^{2n-1} = 1\). The only real number with \(g^{2n-1} = 1\) is \(g = 1\), but \(1\) does not generate \(\mathbb{R}^+\). This is a contradiction. So \(\mathbb{R}^+\) is not cyclic.