7. True or False (If true, then prove it. If false, then give a counterexample.) If every proper subgroup of the group $G$ is abelian, then $G$ is abelian. (Recall that a subgroup $H$ is a proper subgroup if $H \neq G$.)

False. The easiest counterexample is $D_3$ which has six elements $1, \rho, \rho^2, \sigma, \sigma \rho, \sigma \rho^2$ where $\rho$ is rotation by 120°, $\sigma$ is reflection across the x-axis.

$\rho \sigma = \sigma \rho^2$ so $D_3$ is not abelian.

But every subgroup of $D_3$ has order 1, 2, or 3 by Lagrange's theorem and every group of order 1, 2, or 3 is cyclic, hence abelian.