3. Let $G$ be a group and $a \in G$.
   (a) Define "the centralizer of $a". 
   
   The centralizer of the element $a$ in the group $G$ is the set of all elements in $G$ which commute with $a$.

   (b) Prove that the centralizer of $a$ is a subgroup of $G$. 
   
   Closest Take $x$ and $y$ in the centralizer of $a$. So $xa = ax$ and $ya = ay$. We now show that $xy \in$ in centralizer of $a$. Well $(xy)a = x(ya) = x(ay) = (xa)y = (Gx)y = g(xy)$. Thus $xy \in C(a)$.

   (c) Let $G = D_4$ and $a = p$. Find the centralizer of $a$. 
   
   It is clear that $id$, $p$, $p^2$ and $p^3$ all commute with $p$. So all 4 of these elements are in the centralizer of $a$. The centralizer of $p$ is a subgroup of $D_4$, so Lagrange's says that it and divides 8, so the centralizer of $p$ has either 4 or 8 elements. On the other hand $pq = 5p^3 \neq p$ so 0 $\notin C(p)$.

   Thus $C(p) = \langle p \rangle$.

I used $C(a)$ to from the centralizer of $a$. 