5. True or False (If true, then prove it. If false, then give a counterexample.) If $H$ and $K$ are non-zero subgroups of $(\mathbb{Q}, +)$, then the intersection of $H$ and $K$ is non-zero.

There $H$ contains $\frac{a}{b}$ and $K$ contains $\frac{c}{d}$ where $a, b, c, d$ are positive integers.

Observe that $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ acts as $ac \in H$ and $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ acts in $K$.

Thus $ac$, which is not zero, is in $H \cap K$.

6. True or False (If true, then prove it. If false, then give a counterexample.) If $H$ and $K$ are non-zero subgroups of $(\mathbb{R}, +)$, then the intersection of $H$ and $K$ is non-zero.

False. $\mathbb{Z}$ is a non-zero subgroup of $\mathbb{R}$.

$\{n \in \mathbb{Z} \mid n + \frac{1}{2} \}$ is a non-zero subgroup of $\mathbb{R}$.

But $\mathbb{Z} \cap \{n \in \mathbb{Z} \mid n + \frac{1}{2} \} = \emptyset$.

Otherwise, $m = n + \frac{1}{2}$ for some $m, n \in \mathbb{Z}$.

But $\frac{m}{n} = \frac{1}{2}$ is not possible because $\sqrt{2}$ is not rational number.

Hence, the statement is false.