(7)

5. True or False (If true, then prove it. If false, then give a counterexample.) If H and K are non-zero subgroups of $(\mathbb{Q}, +)$, then the intersection of H and K is non-zero.

True. H contains $\frac{a}{b}$ and K contains $\frac{c}{d}$ where a, b, c, d are positive integers.

Observe that $\underbrace{\frac{a}{b}+\cdots+\frac{a}{b}}_{ac \text{ times}} = ac \in H$ and $\underbrace{\frac{c}{d}+\cdots+\frac{c}{d}}_{ad \text{ times}} = ac \in K$

Thus ac, which is not zero, is in $H \cap K$

6. True or False (If true, then prove it. If false, then give a counterexample.) If H and K are non-zero subgroups of $(\mathbb{R}, +)$, then the intersection of H and K is non-zero.

False. $\mathbb{Z}$ is a non zero subgroup of $\mathbb{R}$

$\{n\sqrt{2} \mid n \in \mathbb{Z}\} = \{\ldots, -\sqrt{2}, 0, \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots\}$ is a non-zero subgroup of $\mathbb{R}$

but $\mathbb{Z} \cap \{n\sqrt{2}\} = \{0\}$ otherwise $m = n\sqrt{2}$ for some $m, n \in \mathbb{Z}$.

But $\frac{m}{n} = \sqrt{2}$ is not possible because $\sqrt{2}$ is not a rational number.