3. True or False (If true, then prove it. If false, then give a counterexample.) If $H$ and $K$ are subgroups of the group $G$, then the intersection of $H$ and $K$ is a subgroup of $G$.

**True**

The $x, y \in H \cap K$ which is $xy \in H$ and $xy \in K$ which is $xy \in H \cap K$

**False**

If $e \in H \cap K$

Also $G$ is associative so the operation associates on every subset of $G$.

If $x \in H \cap K$, then $H$ is a group so $x \in H$. Also $x \in K$ and $K$ is a group so $x \in K$.

Thus $x \in H \cap K$.

4. True or False (If true, then prove it. If false, then give a counterexample.) If $H$ and $K$ are subgroups of the group $G$, then the union of $H$ and $K$ is a subgroup of $G$.

**False**

$H \cup K = \{ \emptyset, \emptyset \}$ is not closed, and thus is not a subgroup.