## Math 546, Exam 4, Fall 2004

The exam is worth 50 points.

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Take enough space for each problem. Turn in your solutions in the order: problem 1, problem 2, ... ; although, by using enough paper, you can do the problems in any order that suits you.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office TOMORROW by about 6PM, you may pick it up any time between then and the next class.

I will post the solutions on my website at about 4:00 PM today.

1. (7 points) STATE and PROVE the Chinese Remainder Theorem.

2. (8 points) STATE and PROVE the First Isomorphism Theorem.

3. (7 points) Are the groups $\frac{\mathbb{Z}}{6\mathbb{Z}} \times \frac{\mathbb{Z}}{5\mathbb{Z}}$ and $\frac{\mathbb{Z}}{2\mathbb{Z}} \times \frac{\mathbb{Z}}{15\mathbb{Z}}$ isomorphic? PROVE your answer.

4. (7 points) Are the groups $\frac{\mathbb{Z}}{4\mathbb{Z}} \times \frac{\mathbb{Z}}{4\mathbb{Z}}$ and $\frac{\mathbb{Z}}{4\mathbb{Z}} \times \frac{\mathbb{Z}}{2\mathbb{Z}} \times \frac{\mathbb{Z}}{2\mathbb{Z}}$ isomorphic? PROVE your answer.

5. (7 points) Are the groups $(\mathbb{R}, +)$ and $(\mathbb{R}^{\text{pos}}, \times)$ isomorphic? PROVE your answer. (I am using $\mathbb{R}^{\text{pos}}$ to represent the set of positive real numbers.)

6. (7 points) Let $\phi\colon G_1 \to G_2$ and $\theta\colon G_2 \to G_3$ be group homomorphisms. Prove that $\theta \circ \phi$ is a group homomorphism.

7. (7 points) Suppose that $S$ and $T$ are sets and $\phi\colon S \to T$ and $\theta\colon T \to S$ are functions with $\theta \circ \phi$ equal to the identity function on $S$.
   (a) Does $\phi$ have to be one-to-one? PROVE or give a COUNTEREXAMPLE.
   (b) Does $\theta$ have to be onto? PROVE or give a COUNTEREXAMPLE.