Math 546, Exam 2, Fall, 2004

The exam is worth 50 points.

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Take enough space for each problem. Turn in your solutions in the order: problem 1, problem 2, ... ; although, by using enough paper, you can do the problems in any order that suits you.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office TOMORROW by about 5PM, you may pick it up any time between then and the next class.

I will post the solutions on my website at about 4:00 PM today.

1. (6 points) Define "subgroup". Use complete sentences.

2. (6 points) Define the "center of a group". Use complete sentences.

3. (6 points) STATE Lagrange's Theorem.

4. (7 points) Let G be a finite group with an even number of elements. Prove that there must exist an element $a \in G$ with $a \neq \mathrm{id}$, but $a^2 = \mathrm{id}$.

5. (7 points) Give an example of a finite group G and a proper subgroup H of G, with H not a cyclic group.

6. (6 points) Let G be a group of order pq where p and q are prime numbers. Prove that every proper subgroup of G is cyclic.

7. (6 points) Let g be an element of the group G. Suppose that G has order n. Prove that $g^n = \mathrm{id}$.

8. (6 points) (6 points) Let H be a subgroup of a group. Suppose that $g^{-1}hg \in H$ for all $g \in G$ and $h \in H$. Fix an element $g \in G$. Prove that $gH = Hg$, where gH is the LEFT coset

$$gH = \{gh \mid h \in H\}$$

and Hg is the RIGHT coset

$$Hg = \{hg \mid h \in H\}.$$