Math 546, Exam 3, Spring, 2023

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

No calculators, cell phones, computers, notes, etc.

$\boxed{\text{Make your work correct, complete, and coherent.}}$

The exam is worth 50 points.

The solutions will be posted later today.

(1) (6 points) Define "group homomorphism". Use complete sentences. Include everything that is necessary, but nothing more.

(2) (6 points) Define "cyclic group". Use complete sentences. Include everything that is necessary, but nothing more.

(3) (6 points) Define "normal subgroup". Use complete sentences. Include everything that is necessary, but nothing more.

(4) (6 points) State and prove the First Isomorphism Theorem.

(5) (6 points) Consider $\phi : \frac{\mathbb{Z}}{\langle 3\rangle} \to \frac{\mathbb{Z}}{\langle 6\rangle}$, given by $\phi(a+\langle 3\rangle) = a+\langle 6\rangle$. Is ϕ a group homomorphism? Explain.

(6) (6 points) Consider $\phi : \frac{\mathbb{Z}}{\langle 6\rangle} \to \frac{\mathbb{Z}}{\langle 3\rangle}$, given by $\phi(a+\langle 6\rangle) = a+\langle 3\rangle$. Is ϕ a group homomorphism? Explain.

(7) (7 points) Are the groups $\frac{\mathbb{R}}{\mathbb{Z}}$ and $\frac{\mathbb{R}}{2\mathbb{Z}}$ isomorphic? Explain if the answer is no, and give a proof if the answer is yes. (In this problem, $\mathbb{R}$ is the group of real numbers under addition, $\mathbb{Z}$ is the group of integers under addition, and $2\mathbb{Z}$ is the group of even integers under addition.)

(8) (7 points) Let N be a normal subgroup of the group G. Suppose a, b, c, d are elements of G and that the cosets aN, bN, cN, and dN satisfy

$$aN = bN \quad \text{and} \quad cN = dN.$$

Do the cosets acN and bdN have to be equal? If yes, prove the statement; if no, explain.