

Math 546, Exam 2, Spring 2010

Write everything on the blank paper provided.

You should KEEP this piece of paper.

If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **6** problems.

Write **coherently** in **complete sentences**.

No Calculators or Cell phones.

I will post the solutions later today.

1. (9 points) Let G be a group. Prove that the identity element in G is unique.
2. (7 points) Define *centralizer*. Use complete sentences. Write everything that is necessary for your definition to make sense, but nothing extra.
3. (7 points) Define *order*. Use complete sentences. Write everything that is necessary for your definition to make sense, but nothing extra.
4. (9 points) Let G be an Abelian group and let H be the subset

$$H = \{g \in G \mid g^2 = \text{id}\}$$

of G . Does H have to be a subgroup of G ? If yes, then prove the claim. If no, then give an example.

5. (9 points) Let G be a group and let H be the subset $H = \{g^3 \in G \mid g \in G\}$ of G . Does H have to be a subgroup of G ? If yes, then prove the claim. If no, then give an example.
6. (9 points) List three subgroups of D_4 of order four. (No details are necessary.)