

Math 546, Exam 2, Fall 2011

Write everything on the blank paper provided.

You should KEEP this piece of paper.

If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **8** problems.

Write **coherently** in **complete sentences**.

No Calculators or Cell phones.

I will post the solutions later today.

1. (7 points) Define *centralizer*. Use complete sentences. Write everything that is necessary for your definition to make sense, but nothing extra.
2. (7 points) Define *order*. Use complete sentences. Write everything that is necessary for your definition to make sense, but nothing extra.
3. (6 points) State Lagrange's Theorem.
4. (6 points) Prove Lagrange's Theorem.
5. (6 points) State the result about the relationship between the order of ab , the order of a , and the order of b . Be sure to include all of the hypotheses, but nothing extra.
6. (6 points) Prove the statement in problem 5.
7. (6 points) List 8 subgroups of D_4 in addition to all of D_4 and $\{\text{id}\}$. A small amount of explanation would be perfect. I am thinking of D_4 as the smallest subgroup of $\text{Sym}(\mathbb{C})$ which contains σ and ρ , where $\text{Sym}(\mathbb{C})$ is the group of invertible functions from the complex plane to the complex plane (with operation composition), ρ is rotation counterclockwise by $\pi/2$, and σ is reflection across the x -axis.
8. (6 points) Give an example of a group G and elements a and b in G where a and b each have order 2, but ab has order 10.