Math 546, Exam 2, Spring, 2023

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

No calculators, cell phones, computers, notes, etc.

Make your work correct, complete, and coherent.

The exam is worth 50 points.

The solutions will be posted later today.

(1) (8 points) State and prove Lagrange's Theorem.

(2) (5 points) State Cayley's Theorem.

(3) (8 points) Let H be a subgroup of $(\mathbb{Z}, +)$. Prove that H is a cyclic group. (Please give a complete proof of the result using the notation of $(\mathbb{Z}, +)$. "We proved a more general statement in class" is not an acceptable answer.)

(4) Let H be a subgroup of the group G, let g_0 be a fixed element of G, and

$$H' = \{g_0 h g_0^{-1} \mid h \in H\}.$$

- (a) (5 points) Prove that H' is a subgroup of G.
- (b) (4 points) Exhibit a group isomorphism $\phi : H \to H'$. Prove that your ϕ is an isomorphism.
- (c) (4 points) Give an example of G, H, and H' with $H \neq H'$.

(5) (8 points) Let $(G, *)$ be a group and $H = \{g * g * g \mid g \in G\}$. Is H always a subgroup of G? If yes, prove the result. If no, give a counterexample.

(6) (8 points) List all of the subgroups of U_{12}. Each subgroup should be on your list exactly once. Be sure to explain why you know that you have recorded all of the subgroups.