1. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.)
   If $H$ and $K$ are subgroups of a group $G$, then the intersection $H \cap K$ is also a subgroup of $G$.

2. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.)
   If $H$ and $K$ are subgroups of a group $G$, then the union $H \cup K$ is also a subgroup of $G$.

3. Let $G$ be an abelian group with identity element $e$. Let

   $$H = \{ x \in G \mid x^2 = e \}.$$  

   Prove that $H$ is a subgroup of $G$.

4. Let $G$ be a group with identity element $e$. Suppose that $a$, $b$, and $c$ are elements of $G$ with $c * b * a = e$. Prove that $b * a * c$ is also equal to $e$.

5. Let $\mathbb{R}^*$ represent the set of nonzero real numbers. Define a binary operation $*$ on $\mathbb{R}^*$ by $a * b = b / a$. Is $(\mathbb{R}^*, *)$ a group? If so prove it. If not, show why not.

6. Let $G$ be a group. Let

   $$H = \{ x \in G \mid xy = yx \text{ for all } y \in G \}.$$  

   Prove that $H$ is a subgroup of $G$.

7. Let $G$ be a group with identity element $e$. Suppose that $x^2 = e$ for all $x \in G$. Prove that $G$ is an abelian group.