Solution to the Quiz for March 19, 2003

Express \(v = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\) as a linear combination of

\[
\begin{align*}
  u_1 &= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \\
  u_2 &= \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad \text{and} \\
  u_3 &= \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}.
\end{align*}
\]

(The problem is especially easy if you take advantage of the fact that \(u_1, u_2, u_3\) are an orthogonal set of vectors.) Check your answer!

We solve \(v = c_1 u_1 + c_2 u_2 + c_3 u_3\). Multiply both sides of the equation (on the left) by \(u_1^T\) to see that \(2 = 3c_1\). Multiply by \(u_2^T\) to see that \(-1 = 2c_2\). Multiply by \(u_3^T\) to see that \(1 = 6c_3\). We conclude that

\[
\begin{bmatrix} v \\ 1 \\ 0 \end{bmatrix} = \frac{2}{3} u_1 - \frac{1}{2} u_2 + \frac{1}{6} c_3.
\]

This is correct because:

\[
\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} u_1 - \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} u_2 + \begin{bmatrix} -1 \\ 0 \\ 6 \end{bmatrix} c_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = v.\]