17. (5 points) True or False. (If true, give a proof. If false, give a counter example.)
If \( v_1, v_2, v_3 \) are linearly independent vectors in \( \mathbb{R}^4 \) and \( T: \mathbb{R}^4 \to \mathbb{R}^4 \) is a linear transformation, then \( T(v_1), T(v_2), T(v_3) \) are linearly independent vectors in \( \mathbb{R}^4 \).

**False** Let \( T \) be the linear transformation which sends every vector to \( 0 \).
I can have \( v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \). These are li.

\( T(v_1) = T(v_2) = T(v_3) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \). These are li.d.

18. (5 points) True or False. (If true, give a proof. If false, give a counter example.)
If \( v_1, v_2, v_3 \) are linearly dependent vectors in \( \mathbb{R}^4 \) and \( T: \mathbb{R}^4 \to \mathbb{R}^4 \) is a linear transformation, then \( T(v_1), T(v_2), T(v_3) \) are linearly dependent vectors in \( \mathbb{R}^4 \).

**True** If there are three scalars \( c_1, c_2, c_3 \) not all zero
\( \alpha v_1 + \beta v_2 + \gamma v_3 = 0 \) then
\( T(\alpha v_1 + \beta v_2 + \gamma v_3) = T(0) \)
So \( T(\alpha v_1) + T(\beta v_2) + T(\gamma v_3) = 0 \)
So I have a non-trivial linear combination \( c_1 T(v_1), c_2 T(v_2), c_3 T(v_3) \) which equals 0
\( T(v_1, T(v_2), T(v_3) \) are li.