14. (5 points) Let \( W = \{ f : \mathbb{R} \to \mathbb{R} \mid f \text{ is differentiable} \} \). Is \( W \) a vector space? Explain.

Yes

The sum of two differentiable functions is differentiable.

A scalar times a differentiable function is differentiable.

The function \( f(x) = 0 \) for all \( x \) is differentiable.

15. (5 points) Give an example of three \( 2 \times 2 \) matrices \( A, B, \) and \( C \), with \( A \) not the zero matrix, and \( B \neq C \), but \( BA = CA \).

\[
\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

\[
\begin{bmatrix} 6 & -3 \\ 8 & -4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

Take \( A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \), \( B = \begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \), \( C = \begin{bmatrix} 6 & -3 \\ 8 & -4 \end{bmatrix} \).

We see \( A \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \), \( B \neq C \), but \( BA = CA \).

16. (5 points) Let \( A \) and \( B \) be \( 2 \times 2 \) matrices with \( A \) invertible. Does the column space of \( BA \) have to equal the column space of \( B \)? If the answer is yes, prove it. If the answer is no, give a counterexample.

Yes

\[ \text{Col space } BA \subseteq \text{Col space } B; \]

Every vector in \( \text{Col space } BA \) has the form \( BAx \) for some \( x \).

But \( \text{Col space } B = B(Ax) \), which is in the \( \text{Col space } A \).

\[ \text{Col space } B \subseteq \text{Col space } BA; \]

Every vector in the \( \text{Col space } B \) has the form \( Bx \) for some \( x \).

But \( Bx = BA(A^{-1}x) \), which is in the \( \text{Col space of } BA \).