4. (5 points) Define “linear transformation”. Use complete sentences.

A linear transformation is a function $T$ from the vector space $V$ to the vector space $W$ is if

1. $T(u + v) = T(u) + T(v)$
2. $T(cu) = cT(u)$

for all $u, v \in V$ and $c \in \mathbb{F}$.

5. (5 points) The trace of the square matrix $A$ is the sum of the numbers on its main diagonal. Let $V$ be the set of all $3 \times 3$ matrices with trace 0. The set $V$ is a vector space. You do NOT have to prove this. Give a basis for $V$. NO justification is needed.

I will list 8 linearly independent elements of $V$. $V$ is a proper subspace of the set of all $3 \times 3$ matrices, so I will have listed the entire basis for $V$.

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ M_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ M_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ M_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ M_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ M_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ M_8 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

6. (5 points) Give an example of a matrix which is not diagonalizable. Explain why the matrix is not diagonalizable.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

is not diagonalizable. The only eigenvalue of $A$ is $\lambda = 0$. The eigenspace corresponding to $\lambda = 0$ is spanned by $[1]$. If $A$ were diagonalizable it would have to have two linearly independent eigenvectors. But it doesn't.