9. Let \( v_1, \ldots, v_n \) be \( n \) linearly independent vectors in \( \mathbb{R}^n \). Prove that \( v_1, \ldots, v_n \) is a basis for \( \mathbb{R}^n \).

Let \( A \) be the matrix whose columns are \( v_1, \ldots, v_n \). The columns of \( A \) are linearly independent; hence, \( A \) is invertible by the IMT. Hence, the columns of \( A \) span \( \mathbb{R}^n \) again by the IMT.

10. Let \( A \) and \( B \) be \( 2 \times 2 \) matrices with \( A \) invertible. Does the columns space of \( AB \) have to equal the column space of \( B \)? If the answer is yes, prove it. If the answer is no, give a counterexample.

\[ \text{No} \]

The column space of \( AB \) has nothing to do with the column space of \( B \).

\[ \text{Ex} \]

Take \( A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \). Notice that the column space of \( B \) is the set of all multiples of \( \begin{bmatrix} 2 \\ 4 \end{bmatrix} \).

The product \( AB = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} \). The column space of \( AB \) is spanned by \( \begin{bmatrix} 2 \\ 4 \end{bmatrix} \). These two vector spaces have only the zero vector in common.