PRINT Your Name:  

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. CIRCLE your answer. CHECK your answer whenever possible.

1. Solve the system of equations which corresponds to the following matrix:

\[
\begin{bmatrix}
1 & 2 & 0 & | & 4 \\
0 & 0 & 1 & | & 5 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\]

\[
\begin{align*}
x_1 &= 4 - 2x_2 \\
x_3 &= 5 \\
x_2 & \text{ is arbitrary}
\end{align*}
\]

2. Solve the system of equations which corresponds to the following matrix:

\[
\begin{bmatrix}
1 & 2 & 0 & | & 4 \\
0 & 0 & 1 & | & 5 \\
0 & 0 & 0 & | & 3
\end{bmatrix}
\]

No solution

3. Are the vectors \( v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \), and \( v_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \) linearly independent or linearly dependent? Explain!!

They are \( \text{linearly dependent} \) because \( -2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \)

4. Are the vectors \( v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \), \( v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \), and \( v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \) linearly independent or linearly dependent? Explain!!

They are \( \text{linearly independent} \). Indeed

\[
x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \text{ where row 3 says } x_3 = 0 \]

Row 2 says \( x_2 + x_3 = 0 \) (hence \( x_2 = 0 \)) and row 1 says \( x_1 + x_2 + x_3 = 0 \) (hence \( x_1 = 0 \)).