Math 544, Exam 1, Summer 2006

Write your answers as legibly as you can on the blank sheets of paper provided. Use only one side of each sheet. Leave room on the upper left hand corner of each page for the staple. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

The exam is worth a total of 50 points. There are 7 problems. Problem one is worth 8 points. Each of the other problems is worth 7 points.

SHOW your work. CIRCLE your answer. CHECK your answer whenever possible. No Calculators.

If I know your e-mail address, I will e-mail your grade to you. If I don’t already know your e-mail address and you want me to know it, then send me an e-mail.

If you would like, I will leave your graded exam outside my office door. You may pick it up any time before the next class. If you are interested, be sure to tell me.

I will post the solutions on my website shortly after class is finished.

1. Find the GENERAL solution of the system of linear equations $Ax = b$. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

$$A = \begin{bmatrix}
1 & 4 & 5 & 1 & 8 \\
1 & 4 & 5 & 2 & 10 \\
3 & 12 & 15 & 4 & 26
\end{bmatrix}, \quad x = \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5
\end{bmatrix}, \quad b = \begin{bmatrix}
3 \\
5 \\
11
\end{bmatrix}.$$

2. Consider the system of linear equations.

$$x_1 + ax_2 = 1 \\
ax_1 + 4x_2 = 2.$$

(a) Which values for $a$ cause the system to have no solution?
(b) Which values for $a$ cause the system to have exactly one solution?
(c) Which values for $a$ cause the system to have an infinite number of solutions?

Explain thoroughly.
3. Let \( A \) and \( B \) be \( 2 \times 2 \) matrices with \( AB \) invertible. Does \( A \) have to be invertible? If yes, prove your answer. If no, give a counterexample.

4. Recall that the matrix \( A \) is symmetric if \( A^T = A \). Let \( A \) and \( B \) be \( 2 \times 2 \) symmetric matrices with \( AB = BA \). Does \( AB \) have to be symmetric? If yes, prove your answer. If no, give a counterexample.

5. Let \( v_1, v_2, v_3 \) be linearly independent vectors in \( \mathbb{R}^3 \). Can every vector in \( \mathbb{R}^3 \) be written in terms of \( v_1, v_2, v_3 \) in a unique way? If yes, prove your answer. If no, give a counterexample.

6. Let \( A \) and \( B \) be \( 2 \times 2 \) matrices with \( A \) not equal to the zero matrix and \( BA = A^2 \). Does \( B \) have to equal \( A \)? If yes, prove your answer. If no, give a counterexample.

7. Let \( v_1, v_2, v_3 \) be vectors in \( \mathbb{R}^3 \). Suppose that \( v_1 \) and \( v_2 \) are linearly independent; \( v_1 \) and \( v_3 \) are linearly independent; and \( v_2 \) and \( v_3 \) are linearly independent. Do \( v_1, v_2, v_3 \) have to be linearly independent? If yes, prove your answer. If no, give a counterexample.