Math 544, Summer 2001, Exam 2

PRINT Your Name: _______________________
There are 10 problems on 5 pages. Each problem is worth 5 points. SHOW your
work. [CIRCLE] your answer. CHECK your answer whenever possible.
No Calculators.

1. Define “linear combination”. Use complete sentences.

2. Define “linearly independent”. Use complete sentences.


4. Consider the function \( T: \mathbb{R}^2 \to \mathbb{R}^3 \), which is given by

\[
T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 - 3x_2 \\ x_1 + 4 \\ 5x_2 \end{bmatrix}.
\]

Is \( T \) a linear transformation? If so, then give a matrix \( A \) with \( T(v) = Av \) for all \( v \in \mathbb{R}^2 \). If not, then give an example to show that one of the rules of linear transformation fails to hold.

5. Consider the function \( T: \mathbb{R}^2 \to \mathbb{R}^3 \), which is given by

\[
T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 - 3x_2 \\ x_1 + 4x_2 \\ 5x_2 \end{bmatrix}.
\]

Is \( T \) a linear transformation? If so, then give a matrix \( A \) with \( T(v) = Av \) for all \( v \in \mathbb{R}^2 \). If not, then give an example to show that one of the rules of linear transformation fails to hold.

6. The function \( T: \mathbb{R}^2 \to \mathbb{R}^3 \) is a linear transformation with \( T \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \) and \( T \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \). Find a matrix \( A \) with \( T(v) = Av \) for all \( v \in \mathbb{R}^2 \).

7. True or False. (If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE.)
Let \( A \), \( B \), and \( C \) be \( 2 \times 2 \) matrices with \( A \) not equal to the zero matrix. If 
\( AB = AC \), then \( B = C \).

8. True or False. (If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE.)
If \( v_1, v_2, v_3, v_4 \) are in \( \mathbb{R}^4 \) and \( v_3 \) is not a linear combination of \( v_1, v_2, v_4 \), then 
\( \{v_1, v_2, v_3, v_4\} \) is linearly independent.
9. Are
\[ v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \; v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \; v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \; \text{and} \; v_4 = \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix} \]
linearly dependent or linearly independent? Show your work. Check your answer.

10. Find the general solution of the following system of linear equations:
\[
\begin{align*}
    x_1 &+ x_2 &- x_5 &= 1 \\
    x_1 &+ 2x_2 &+ 2x_3 &+ x_4 &+ 2x_5 &= 2 \\
    x_1 &- x_3 &+ x_4 &+ x_5 &= 0.
\end{align*}
\]
Also find three particular solutions of this system of equations. Be sure to check that all three of your particular solutions really satisfy the original system of linear equations.