Math 544  
Exam 2  
Spring 2003

PRINT Your Name: ________________________________

Please also write your name on the back of the exam.

There are 8 problems on 4 pages. Problem 4 is worth 15 points. Each of the other problems is worth 5 points. The exam is worth a total of 50 points. SHOW your work. [CIRCLE] your answer. CHECK your answer whenever possible. No Calculators.

If I know your e-mail address, I will e-mail your grade to you. If I don’t already know your e-mail address and you want me to know it, send me an e-mail.

I will leave your exam outside my office door about 6PM today, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the exam is finished.

1. Define “linearly independent”. Use complete sentences.

2. Define “non-singular”. Use complete sentences.

3. Let $A$ be an $n \times n$ matrix. List three conditions which are equivalent to the statement “$A$ is non-singular”. (I expect three new conditions in addition to “$A$ is non-singular”. Also, I do not expect you to repeat your answer to problem 2.)

4. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$.

   (a) Find the GENERAL solution of the system of equations $Ax = b$.

   (b) List two SPECIFIC solutions of $Ax = b$, if possible. CHECK that the specific solutions satisfy the equations.

   (c) Are the columns of $A$ linearly independent? Explain.

   (d) List vectors $v_1, \ldots, v_r$ so that the null space of $A$ is the span of $v_1, \ldots, v_r$. (You pick the appropriate number for $r$.)

   (e) Is $b$ in the column space of $A$? Explain.
5. True or False. (If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE.) If $U$ and $V$ are subspaces of $\mathbb{R}^2$, then the union $U \cup V$ is also a subspace of $\mathbb{R}^2$.

6. Let $v_1$, $v_2$, and $v_3$ be non-zero vectors in $\mathbb{R}^4$. Suppose that $v_i^Tv_j = 0$ for all subscripts $i$ and $j$ with $i \neq j$. Prove that $v_1$, $v_2$, and $v_3$ are linearly independent.

7. Let

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4 \left| \begin{array}{cccc} 1 & 2 & 3 & 4 \\ 2 & x_1 & 3 & 4 \\ x_1 & +3x_2 & +4x_3 & = 2x_4 \end{array} \right. \right\}.$$

Is $W$ a vector space? Explain.

8. Let

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \left| x_2 = x_1 x_3 \right. \right\}.$$

Is $W$ a vector space? Explain.