Math 544, Exam 1, Spring 2003

PRINT Your Name:__________________________________________

Please also write your name on the back of the exam.
There are 10 problems on 6 pages. Each problem is worth 5 points. SHOW your work. [CIRCLE] your answer. CHECK your answer whenever possible. No Calculators.

If I know your e-mail address, I will e-mail your grade to you.
I will leave your exam outside my office door about 6PM today, you may pick it up any time between then and the next class.
I will post the solutions on my website shortly after the exam is finished.

1. Find the general solution of the following system of linear equations. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

\[
\begin{align*}
2x_1 + 3x_2 &= 4 \\
4x_1 + 6x_2 &= 7
\end{align*}
\]

2. Find the GENERAL solution of the following system of linear equations. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

\[
\begin{align*}
x_1 + 3x_2 + 3x_4 &= 1 \\
x_1 + 3x_2 + x_3 + 5x_4 &= 3 \\
2x_1 + 6x_2 + x_3 + 8x_4 &= 4.
\end{align*}
\]

3. Express \( b = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \) as a linear combination of \( v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \) and \( v_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \).


6. STATE the theorem about linear dependence or independence of $p$ vectors in $\mathbb{R}^m$. (I call this theorem the Short Fat Theorem.)

7. True or False. (If true, explain why or give a proof. If false, give a counter example.) If $A$, $B$, and $C$ are $2 \times 2$ matrices with $A \neq 0$ and $BA = CA$, then $B = C$.

8. True or False. (If true, explain why or give a proof. If false, give a counter example.) If $A$, $B$ are $2 \times 2$ symmetric matrices, then $AB$ is a symmetric matrix.

9. Which numbers $a$ make $A = \begin{bmatrix} 1 & 2 \\ 2 & a \end{bmatrix}$ non-singular? Explain.

10. Let $v_1$, $v_2$, $v_3$ be linearly independent vectors. PROVE that the vectors $v_1 + v_2$, $v_1 - v_2$, $v_3$ are also linearly independent.