1. (5 points) Define “closed under addition”. Use complete sentences. Include everything that is necessary, but nothing more.

2. (5 points) Define “basis”. Use complete sentences. Include everything that is necessary, but nothing more.

3. (5 points) Let $A$ be an $n \times n$ matrix and let $W = \{v \in \mathbb{R}^n | Av = 2v\}$. Is $W$ a subspace of $\mathbb{R}^n$? If yes, then give a complete, correct, proof. If no, then give an explicit example that shows that $W$ is not a subspace of $\mathbb{R}^n$.

4. (5 points) Let $U$ and $V$ be subspaces of $\mathbb{R}^n$. Is the union $U \cup V$ a subspace of $\mathbb{R}^n$? If yes, then give a complete, correct, proof. If no, then give an explicit example that shows that $U \cup V$ is not a subspace of $\mathbb{R}^n$.

5. (10 points) State the four theorems about dimension.

6. (10 points) Let $V$ and $W$ be subspaces of $\mathbb{R}^n$ with $V \subseteq W$.
   (a) Does the dimension of $V$ have to be $\leq$ the dimension of $W$? If yes, then give a complete, correct, proof. If no, then give an explicit example.
   (b) Suppose $\dim V = \dim W$. Does $V$ have to equal $W$? If yes, then give a complete, correct, proof. If no, then give an explicit example.
7. (10 points) Let $A$ and $B$ be $n \times n$ matrices, with $A$ non-singular. Answer each question. If the answer is yes, then give a complete, correct, proof. If the answer is no, then give an example.

(a) Does the null space of $B$ have to be equal the null space of $AB$?
(b) Does the dimension of the null space of $B$ have to equal the dimension of the null space of $AB$?
(c) Does the column space of $B$ have to equal the column space of $AB$?
(d) Does the dimension of the column space of $B$ have to equal the dimension of the column space of $AB$?