PRINT Your Name: __________________________

There are 9 problems on 4 pages. Problem 1 is worth 20 points. Each of the
other problems is worth 10 points. SHOW your work. CIRCLE your answer.
CHECK your answer whenever possible. No Calculators.

1. Let

\[ A = \begin{bmatrix}
1 & 1 & 0 \\
1 & 0 & 1 \\
1 & -1 & 0 \\
1 & 0 & -1
\end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix}
5 \\
2 \\
1 \\
4
\end{bmatrix}. \]

(It might be to your advantage to notice that the columns of \( A \) form an
orthogonal set.)

(a) Find a matrix \( B \) so that \( BA \) is equal to the \( 3 \times 3 \) identity matrix.

(b) Solve \( Ax = b \).

2. Define “column space”. Use complete sentences.


4. True or False. If the statement is true, then PROVE the statement. If the
statement is false, then give a COUNTEREXAMPLE. If \( A \) and \( B \) are \( 2 \times 2 \)
nonsingular matrices, then \( A + B \) is a nonsingular matrix.

5. True or False. If the statement is true, then PROVE the statement. If the
statement is false, then give a COUNTEREXAMPLE. If \( U \) and \( V \) are
subspaces of \( \mathbb{R}^n \), then the intersection of \( U \) and \( V \) is also a subspace of \( \mathbb{R}^n \).

6. Let \( W \) be the subspace of \( \mathbb{R}^4 \) which is spanned by

\[ w_1 = \begin{bmatrix}
0 \\
1 \\
0 \\
0
\end{bmatrix}, \quad w_2 = \begin{bmatrix}
1 \\
1 \\
2 \\
1
\end{bmatrix}, \quad w_3 = \begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix}. \]

Find an orthogonal set which forms a basis for \( W \).

7. Is

\[ \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mid x_1x_2 = x_3 \right\} \]

a vector space? If so, explain why. If not, give an example to show that one of
the rules of vector space fails to hold.

8. Is the function \( F: \mathbb{R}^2 \to \mathbb{R}^2 \), which is defined by

\[ F \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix}
x_1^2 \\
x_1x_2
\end{bmatrix}, \]

a linear transformation? If so, explain why. If not, give an example to show
that one of the rules of linear transformation fails to hold.
9. Is the function $F: \mathbb{R}^2 \to \mathbb{R}^3$, which is defined by

$$F \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix},$$

a linear transformation? If so, explain why. If not, give an example to show that one of the rules of linear transformation fails to hold.