MATH 544, 1997, EXAM 1

PRINT Your Name: _____________________________

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. [CIRCLE] your answer. CHECK your answer whenever possible.

1. Solve the system of equations which corresponds to the following matrix:

\[
\begin{bmatrix}
1 & 2 & 0 & 4 \\
0 & 0 & 1 & 5 \\
0 & 0 & 0 & 0
\end{bmatrix}.
\]

2. Solve the system of equations which corresponds to the following matrix:

\[
\begin{bmatrix}
1 & 2 & 0 & 4 \\
0 & 0 & 1 & 5 \\
0 & 0 & 0 & 3
\end{bmatrix}.
\]

3. Are the vectors \( v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \), and \( v_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \) linearly independent or linearly dependent? Explain!!

4. Are the vectors \( v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \), \( v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \), and \( v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \) linearly independent or linearly dependent? Explain!!

5. Solve the following system of equations:

\[
x_1 + 3x_2 + 7x_3 = 28 \\
2x_1 + 7x_2 + 16x_3 = 64 \\
3x_1 + 11x_2 + 26x_3 = 103.
\]

6. Find the inverse of

\[
A = \begin{bmatrix}
2 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 1
\end{bmatrix}.
\]

7. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If \( A \) and \( B \) are \( 2 \times 2 \) symmetric matrices, then \( AB \) is a symmetric matrix.

8. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If \( A \) and \( B \) are \( 2 \times 2 \) nonsingular matrices, then \( AB \) is a nonsingular matrix.

9. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If \( A \) and \( B \) are \( 2 \times 2 \) nonsingular matrices, then \( A + B \) is a nonsingular matrix.

10. Define “linearly independent”.