Math 242, Spring 1994, Final Exam

SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK your answers**. Each problem is worth 15 points.

1. Find one nontrivial solution of $tx'' - 2x' + tx = 0$ with $x(0) = 0$.

2. Solve the initial value problem $x'' + 5x' + 4x = f(t)$, $x(0) = x'(0) = 0$, where

$$f(t) = \begin{cases} 1 & \text{if } t < 3 \\ 0 & \text{if } 3 \leq t. \end{cases}$$

3. State the Existence and Uniqueness Theorem for SECOND order LINEAR differential equations.

4. A pitcher of buttermilk initially at 30 degrees is to be cooled by setting it on the front porch, where the temperature is 0 degrees. Suppose that the temperature of the buttermilk has dropped to 20 degrees after 20 minutes. Assume that the temperature of the buttermilk obeys Newton's law of cooling, which states that

$$\frac{dT}{dt} = k(A - T),$$

   where $T(t)$ is the temperature of a body immersed in a medium of constant temperature A and k is a constant. When will the temperature of the buttermilk reach 5 degrees?

5. Solve the initial value problem $y'' + 4y' + 3y = 16e^x$, $y(0) = 3$, $y'(0) = -3$.

6. Find the general solution of $y'' + 25y = \sin 5x$.

7. Find the general solution of $x^2y'' - 4xy' + 6y = x^3$.

8. One solution of $xy'' + (x - 1)y' - y = 0$ is $y = e^{-x}$. Find the general solution of the differential equation.

9. Find the general solution of $xy' + (2x - 3)y = 4x^4$.

10. Find the general solution of $2xyy' = 4x^2 + 3y^2$.