

### Math 242, 1993, Final Exam

There are 10 problems. Each problem is worth 15 points. SHOW your work. **CIRCLE** your answer. CHECK your answers.

1. State the Existence and Uniqueness Theorem about linear differential equations of second order.
2. Find ALL solutions of  $y'' - 5y' + 6y = e^x$ .
3. Find ALL solutions of  $x^2y'' - 3xy' + 3y = 0$ .
4. Find ALL solutions of  $xy' = 3y + x^4 \cos x$ , with  $y(2\pi) = 1$ .
5. Find ALL solutions of  $x(x + y)y' + y(3x + y) = 0$ .
6. A 120-gallon tank initially contains 90 lb of salt dissolved in 90 gal. of water. Brine containing 2 lb/gal of salt flows into the tank at the rate of 4 gal/min, and the mixture flows out of the tank at the rate of 3 gal/min. How much salt does the tank contain when it is full?
7. Find ALL solutions of  $y'' + 9y = 2 \sec 3x$ .
8. USE THE METHOD OF LAPLACE TRANSFORMS to find one nontrivial solution of  $tx'' + (3t - 1)x' + 3x = 0$  with  $x(0) = 0$ .
9. USE THE METHOD OF LAPLACE TRANSFORMS to solve  $x'' + 5x' + 4x = f(t)$ , with  $x(0) = 0$  and  $x'(0) = 0$ , where
$$f(t) = \begin{cases} 1 & \text{if } t < 2, \text{ and} \\ 0 & \text{if } 2 < t. \end{cases}$$
10. USE POWER SERIES to solve  $y'' + xy' + y = 0$ .