

Math 242, 1990, Exam 1

There are 4 problems. Use your own paper. SHOW your work. CIRCLE your answer. Each problem is worth 25 points.

1. Solve $2xyy' = x^2 + 2y^2$.

2. Consider the Initial Value Problem

$$(*) \quad (1 + x^2)y' = (1 + y)^2 \quad y(a) = b.$$

- For which values of a and b does the Existence and Uniqueness Theorem guarantee that (*) has a unique solution.
- Solve (*) for $a = b = 0$.
- Solve (*) for $a = 0$ and $b = -1$.

3. A 400 gallon tank initially contains 100 gallons of brine containing 50 pounds of salt. Brine containing 1 pound of salt per gallon enters the tank at the rate of 5 gallons per second, and the mixed brine flows out at the rate of 3 gallons per second. How much salt will be the tank contain when it is full of brine?

4. Solve the Initial Value Problem

$$y'' + y = 0 \quad y(0) = 2 \quad \text{and} \quad y'(0) = 3.$$