Suppose the velocity of a motorboat coasting in water satisfies the differential equation \( \frac{dv}{dt} = kv^2 \). The initial speed of the motorboat is \( v(0) = 10 \) meters per second (m/s), and \( v \) is decreasing at the rate 1 m/s\(^2\) when \( v = 5 \) m/s. How long does it take for the velocity of the boat to decrease to 1 m/s?

**ANSWER:** We separate the variables and integrate to see that \( \int \frac{dv}{v^2} = \int kdt \); so, \(-1/v = kt + C\). Plug in \( t = 0 \) to learn that \(-1/10 = C\). Let \( t_1 \) be the time when \( v = 5 \). We are told that at time \( t_1 \), we have \( \frac{dv}{dt}(t_1) = -1 \). Plug \( t_1 \) into \( \frac{dv}{dt} = kv^2 \) to learn:

\[
-1 = \frac{dv}{dt}(t_1) = kv(t_1)^2 = k(25).
\]

So, \( k = -1/25 \). Thus,

\[
v = \frac{-1}{kt + C} = \frac{-1}{\frac{1}{25}t + \frac{1}{10}}.
\]

Multiply top and bottom by \(-50\) to get

\[
v = \frac{50}{2t + 5}.
\]

We find the time \( t_2 \), when \( v(t_2) = 1 \):

\[
1 = \frac{50}{2t_2 + 5}
\]

\[
2t_2 + 5 = 50
\]

\[
2t_2 = 45
\]

\[
t_2 = 22.5 \text{ seconds}
\]