

Math 242, Final Exam, Summer 2012

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 100 points. There are **7** problems. **SHOW** your work.

CIRCLE your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones.

1. (16 points) Solve $y'' - 3y' + 2y = 3e^{-x} - 10 \cos 3x$, $y(0) = 1$, $y'(0) = 2$. Express your answer in the form $y(x)$. **CHECK** your answer.

2. (14 points) Consider a body that moves horizontally through a medium whose resistance is proportional to the square of the velocity v , so that $\frac{dv}{dt} = -kv^2$, for some positive constant k . Let v_0 be the velocity of the object at time 0 and x_0 be the position of the object at time 0.
 - (a) Find the velocity $v(t)$ of the object at time t . **CHECK** your answer.
 - (b) Find the position $x(t)$ of the object at time t . **CHECK** your answer.
 - (c) Find $\lim_{t \rightarrow \infty} x(t)$.

3. (14 points) Solve $xy + y^2 - x^2y' = 0$. Express your answer in the form $y(x)$. **CHECK** your answer.

4. (14 points) Solve $xy' + 2y = 6x^2\sqrt{y}$. Express your answer in the form $y(x)$. **CHECK** your answer.

5. (14 points) Find $\mathcal{L}^{-1}\left(\frac{1}{s(s^2+4)}\right)$.

6. (14 points) Find $\mathcal{L}^{-1}\left(\frac{s}{(s^2+1)^3}\right)$.

7. (14 points) Find a nontrivial solution of $tx'' + (4t - 2)x' + (13t - 4)x = 0$ such that $x(0) = 0$. Express your answer in the form $x(t)$. **CHECK** your answer.