Math 242,   Final Exam   Spring, 2021

Write everything on the blank paper that you brought. There should be nothing on your desk except this exam, the blank paper that you brought, and a pen or pencil. When you are finished, send a picture of your solutions to

   kustin@math.sc.edu

ALSO, LEAVE A PHYSICAL COPY OF YOUR SOLUTIONS WITH ME. Fold your solutions in half and write your name on the outside.

The exam is worth 100 points. There are 6 problems. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

   IF YOU LEAVE THE ROOM DURING THE EXAM, PLEASE LEAVE YOUR PHONE WITH ME BEFORE YOU GO.

(1) (16 points) A 1000 gallon holding tank that catches runoff from some chemical process initially has 400 gallons of water with 100 pounds of pollution dissolved in it. Polluted water flows into the tank at a rate of 4 gal/hr and contains 2 pounds/gal of pollution in it. A well mixed solution leaves the tank at 2 gal/hr. **Give an Initial Value Problem for the number of pounds of pollution in the tank at time t hours. Do not solve the Initial Value Problem.**

(2) (16 points) Find $\mathcal{L}(\cos^2(2t))$.

(3) (17 points) Find the general solution of $y' - y = y^2 e^x$. Put your answer in the form $y = y(x)$. **Please check your answer.**

(4) (17 points) Find the general solution of $y'' + 4y' + 4y = e^{2x}$. Put your answer in the form $y = y(x)$. **Please check your answer.**

(5) (17 Points) Find the general solution of $xy\frac{dy}{dx} = x^2 + y^2$. Put your answer in the form $y = y(x)$. **Please check your answer.**

(6) (17 points) Use Laplace transforms to solve the Initial Value Problem $x'' - 10x' + 9x = 5t$, with $x(0) = -1$ and $x'(0) = 2$. Put your answer in the form $x = x(t)$. **Please check your answer.**