Math 242, Final Exam, Fall 2012

Write everything on the blank paper provided. If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 100 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

Nothing may be on your desk except things that came from me. In particular, **no Calculators or Cell phones** may be on your desk.

Your work must be coherent and correct.

1. (12 points) Find the general solution of $y^{(3)} + y'' = 3e^x + 4x^2$. (In this problem, y is a function of x.)

2. (11 points) Find $\mathcal{L}^{-1}(\arctan \frac{1}{s})$. (In this problem, x is a function of t.)

3. (11 points) Find $\mathcal{L}^{-1}(\frac{2}{(s-1)(s^2+4)})$. (In this problem, x is a function of t.)

4. (11 points) Find a nontrivial solution of $tx'' - 2x' + tx = 0$ with $x(0) = 0$. (In this problem, x is a function of t.)

5. (11 points) The motion of an object with position $x(t)$ and velocity $v(t)$ is described by the initial value problem $\frac{dv}{dt} = -kv^{3/2}$, $x(0) = x_0$ and $v(0) = v_0$, for some positive constant k. What is the velocity of the object at time t? What is the position of the object at time t? What is $\lim_{t\to\infty} x(t)$?

6. (11 points) Solve the initial value problem $\frac{dx}{dt} = x(x^2 - 4)$, $x(0) = x_0$. Draw some of the solutions for $t \geq 0$, depending on the value of x_0.

7. (11 points) Consider two tanks. The first tank has a volume of 100 gals. of brine. The second tank has a volume of 200 gals. of brine. Each tank initially contains 50 lbs. of salt. Pure water flows into the first tank at the rate of 5 gal./min. The well mixed solution flows out of tank 1 and into tank 2 at the rate of 5 gal./min. The well mixed solution flows out of tank 2 at the rate of 5 gal./min.
   (a) How much salt is in the first tank at time t?
   (b) How much salt is in the second tank at time t?

8. (11 points) Solve $3y + x^3y^4 + 3xy' = 0$. (In this problem, y is a function of x.)

9. (11 points) Solve $2xy + x^2y' = y^2$. (In this problem, y is a function of x.)