

## Math 242, Fall 1994, Final Exam

PRINT Your Name: \_\_\_\_\_

There are 15 problems on 9 pages. Problem 1 is worth 16 points each. Each of the other problems is worth 6 points. The exam is worth a total of 100 points. SHOW your work. **CIRCLE** your answer. **CHECK** your answer, whenever possible.

1.
  - (a) State the Existence and Uniqueness Theorem for first order differential equations.
  - (b) What does the Existence and Uniqueness Theorem tell you about the Initial Value Problem

$$(1 + x^2)y' = (y - 3)^2 \quad y(0) = 3?$$

- (c) Solve the Initial Value Problem of part (b).
2. A 400 gallon tank initially contains 200 gallons of brine containing 30 pounds of salt. Brine containing 3 pounds of salt per gallon enters the tank at the rate of 6 gal./sec., and the mixed brine in the tank flows out at the rate of 2 gal./sec.. How much salt will the tank contain at the moment it becomes full?
  3. Find all solutions of  $x^2y' = xy + x^2e^{y/x}$ .
  4. Find all solutions of  $xy' + (2x - 3)y = 4x^4$ .
  5. Find all solutions of  $y'' + 2y' + y = 0$ .
  6. Find all solutions of  $y'' - 4y' + 13y = 0$ .
  7. Find all solutions of  $y'' - 3y' + 2y = 10 \sin x$ .
  8. Find all solutions of  $y'' - 3y' + 2y = e^x$ .
  9. Find all solutions of  $x^2y'' - 4xy' + 6y = 0$ .
  10. Find all solutions of  $y' = \sin 2x \cos 3x$ .
  11. Find all solutions of  $y'' + y = \tan x$ .
  12. Find  $\mathcal{L}^{-1} \left( \ln \frac{s-3}{s+2} \right)$ .

13. Find  $\mathcal{L}^{-1}\left(\frac{5-2s}{s^2+7s+10}\right)$ .

14. Find one nontrivial solution of

$$\begin{cases} tx'' - 2x' + tx = 0, \\ x(0) = 0. \end{cases}$$

15. Solve

$$\begin{cases} x'' + 4x = \begin{cases} t & \text{if } 0 \leq t < 1, \\ 0 & \text{if } 1 \leq t, \end{cases} \\ x(0) = 0, \quad \text{and} \quad x'(0) = 0. \end{cases}$$