

**Math 242, Fall 1994, Exam 1**

PRINT Your Name: \_\_\_\_\_

There are 7 problems on 4 pages. Problem 2 is worth 8 points. Each of the other problems is worth 7 points. The exam is worth a total of 50 points. SHOW your work. *CIRCLE* your answer.

1. State the Existence and Uniqueness Theorem for first order differential equations.
2.
  - (a) What does the Existence and Uniqueness Theorem tell you about the Initial Value Problem

$$(1 + x^2)y' = (7 + y)^2 \quad y(0) = -7?$$

- (b) Solve the Initial Value Problem of part (a).

3. Solve the Initial Value Problem  $y' = x \cos x$ ,  $y(0) = 0$ .
4. Solve  $xy' = y + 2\sqrt{xy}$ .
5. Solve  $y' = y + y^3$ .
6. Solve the initial value problem

$$\begin{cases} y'' - 9y = 0 \\ y(0) = -1 \\ y'(0) = 9 \end{cases}$$

If it helps you, you may use the fact that  $y_1(x) = e^{3x}$  and  $y_2(x) = e^{-3x}$  both are solutions of the differential equation  $y'' - 9y = 0$ .

7. A 400 gallon tank initially contains 100 gallons of brine containing 25 pounds of salt. Brine containing 2 pounds of salt per gallon enters the tank at the rate of 5 gal./sec., and the mixed brine in the tank flows out at the rate the rate of 4 gal./sec.. How much salt will the tank contain at the moment it becomes full?