Math 242, Exam 3, Summer 2012

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **6** problems. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones.

1. (7 points) Solve $y'' + 2y' - 3y = e^x$. Express your answer in the form $y(x)$. **Check your answer.**

2. (7 points) Solve $y''' - 5y'' + 8y' - 4y = 0$. Express your answer in the form $y(x)$. **Check your answer.**

3. (6 points) Solve $y''+y'+y = 0$. Express your answer in the form $y(x)$. **Check your answer.**

4. (6 points) Suppose that a body moves through a resisting medium with resistance proportional to its velocity $v(t)$, so that $\frac{dv}{dt} = -kv$ for some positive constant k . Let $x(t)$ be the position of the object at time t . Let $v(0) = v_0$ and $x(0) = x_0$. Find the velocity and position of the object at time t . Find $\lim\limits_{t\to\infty} x(t)$.

Please turn over.

5. (6 points) Consider a population $P(t)$ which satisfies the Differential Equation

$$\frac{dP}{dt} = aP^2 - bP, \tag{1}$$

where a and b are positive constants. Let $B(t) = aP(t)^2$ and $D(t) = bP(t)$. Call $B(t)$ the birth rate at time t and $D(t)$ the death rate at time t. When we first thought about population models we learned that $P(t) = 0$ and $P(t) = M$ are equilibrium solutions for the Differential Equation

$$\frac{dP}{dt} = kP(P - M), \tag{2}$$

where k and M are positive constants. We also learned that the solution of (2) is

$$P(t) = \frac{MP(0)}{P(0) + (M - P(0))e^{kMt}}.$$

(a) Express the "M" of (2) in terms of the data $B(0)$, $D(0)$, and $P(0)$ from (1).

(b) Suppose that some population is modeled by (1) and that the initial population is 100 and there are 10 births per month and 9 deaths per month occurring at $t = 0$. How many months does it take until $P(t)$ reaches 10 times the threshold population M?

6. (6 points) Consider the initial value problem $\frac{dy}{dx} = x + y^3$, $y(1) = 2$. Use Euler's method to approximate $y(3/2)$. Use two steps, each of size $1/4$.