Math 242, Exam 2, Summer 2012

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **5** problems. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones.

1. Solve $(x+y)\frac{dy}{dx} = x - y$. Express your answer in the form $y(x)$. **Check your answer.**

2. Solve $\frac{dy}{dx} = (4x+y)^2$. Express your answer in the form $y(x)$. **Check your answer.**

3. Solve $x\frac{dy}{dx} + 6y = 3xy^{4/3}$. Express your answer in the form $y(x)$. **Check your answer.**

4. Consider two tanks. The first tank has a volume of 100 gals. of brine. The second tank has a volume of 200 gals. of brine. Each tank initially contains 50 lbs. of salt. Pure water flows into the first tank at the rate of 5 gal./min. The well mixed solution flows out of tank 1 and into tank 2 at the rate of 5 gal./min. The well mixed solution flows out of tank 2 at the rate of 5 gal./min.
   (a) How much salt is in the first tank at time t?
   (b) How much salt is in the second tank at time t?

5. Consider the Differential Equation $\frac{dx}{dt} = -(3-x)^2$.
   (a) Find all equilibrium solutions $x(t) = x_e$ for all t for some constant x_e.
   (b) For each equilibrium solution $x(t) = x_e$ of the DE, answer the following questions:
   (i) If $x(0)$ is a little less than x_e, does the corresponding solution $x(t)$ head toward or away from the equilibrium solution $x = x_e$.
   (ii) If $x(0)$ is a little more than x_e, does the corresponding solution $x(t)$ head toward or away from the equilibrium solution $x = x_e$.
   (c) Sketch a few solutions of the DE.
   (d) Solve the DE.