Math 242,   Exam 2,   Spring 2010

Write everything on the blank paper provided.

You should KEEP this piece of paper.

If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **5** problems. Each problem is worth 10 points.

SHOW your work. $\boxed{CIRCLE}$ your answer. Write **coherently**.

No Calculators or Cell phones.

I will post the solutions later today.

1. Consider the Initial Value Problem $\frac{dx}{dt} = (x-1)(x-4)$, $x(0) = x_0$.
   (a) Solve the Initial Value Problem.
   (b) Draw some of the solutions.
   (c) Which choices for x_0 cause x to go to infinity at some finite time?
   (d) Which choices for x_0 cause x to go toward a finite constant as t goes to infinity.

2. An object is moving in a straight line with position at time t given by $x(t)$ and velocity at time t given by $v(t)$. The object's motion satisfies the initial value problem

$$\frac{dv}{dt} = -kv^{3/2}, \quad v(0) = v_0, \quad \text{and} \quad x(0) = x_0,$$

   where k is a constant. Find $\lim_{t\to\infty} x(t)$.

3. Consider the initial value problem $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$. Use Euler's method to approximate $y(2/10)$. Use two steps, each of size $1/10$.

4. Solve $x\frac{dy}{dx} + 6y = 3xy^{4/3}$. Express your answer in the form $y(x)$. Check your answer.

5. Solve $(x^2 + 1)\frac{dy}{dx} + 3xy = 6x$. Express your answer in the form $y(x)$. Check your answer.