Math 242,   Exam 2,   Spring 2017,   1:15 Class

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today. The exams will be returned in class on Tuesday, Feb. 28.

No Calculators or Cell phones.

(1) (a) State the Existence and Uniqueness Theorem for second order linear Differential Equations.

   (b) What does what does (a) tell you about the Initial Value Problem

$$e^x \frac{d^2y}{dx^2} + \frac{1}{x-3} \frac{dy}{dx} = x, \quad y(1) = -1, \quad y'(1) = 6?$$

   (Please explain your answer.)

(2) A tank with 200 gallons of brine solution contains 40 lbs of salt. A brine solution with 2 pounds of salt per gallon of solution is pumped into the tank at a rate of 4 gal/min. The well mixed solution is pumped out of the tank out at a rate of 4 gal/min. How much salt is in the tank after 1 hour? How much salt is in the tank after a very long time? SET UP THE INITIAL VALUE PROBLEM. DO NOT SOLVE THE INITIAL VALUE PROBLEM.

(3) Solve $xy\frac{dy}{dx} + 4x^2 + y^2 = 0$. Express your answer in the form $y = y(x)$. Please check your answer.

(4) Solve $\frac{dy}{dx} = (y-1)(y-3)$. Draw some of the solutions of this Differential Equation for various values of $y(0)$.

(5) Find all constants r for which $y = e^{rx}$ a solution of $y'' + 3y' + 2y = 0$. Find a constant A with $y = Ae^{3x}$ a solution of $y'' + 3y' + 2y = e^{3x}$. What is the general solution of $y'' + 3y' + 2y = e^{3x}$?