Math 242,   Exam 2,   Fall 2016

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

No Calculators or Cell phones.

(1) Solve $x\dfrac{dy}{dx} + 6y = 3xy^{4/3}$. Write your answer in the form $y = y(x)$. Check your answer.

(2) Solve $\dfrac{dx}{dt} = 7x - x^2 - 10$. Sketch a few solutions.

(3) A motor boat is moving at 40 feet per second when its motor suddenly quits and 10 seconds later the boat has slowed to 20 feet/second. The only force acting on the boat is resistance and resistance is proportional to velocity. How far will the boat coast in all?

(4) A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved in it. Water enters the tank at a rate of 9 gal/hr and the water entering the tank has a salt concentration of $\frac{1}{5}(1+\cos t)$ lbs/gal. If a well mixed solution leaves the tank at a rate of 6 gal/hr, how much salt is in the tank at time t? **Set up the initial value problem. You do not have to solve it.**

(5) Consider the differential equation

$$y'' - 5y' + 6y = e^{-x}. \tag{1}$$

(a) Which of the functions $y_1 = e^{-x}$, $y_2 = e^{2x}$, $y_3 = e^{3x}$ is a solution of the corresponding homogeneous problem

$$y'' - 5y' + 6y = 0?$$

(b) Find a constant α and a function y_i (selcted from $y_1 = e^{-x}$, $y_2 = e^{2x}$, $y_3 = e^{3x}$) so that $y = \alpha y_i$ is a solution of the original differential equation (1).

(c) What is the general solution of the original differential equation (1)?