Math 242, Exam 1, Summer 2012

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **5** problems. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones.

1. Solve $2x\frac{dy}{dx} = y + x + 1$. Express your answer in the form $y(x)$. **Check your answer.**

2. Solve $2y\frac{dy}{dx} = \frac{x}{\sqrt{x^2-16}}$, $y(5) = 2$. Express your answer in the form $y(x)$. **Check your answer.**

3. A tank initially contains 100 gallons of brine in which 50 pounds of salt are dissolved. A brine containing 2 pounds per gallon of salt runs into the tank at the rate of 5 gallons per minute. The mixture is kept uniform by stirring and flows out of the tank at the rate of 4 gallons per minute. How many pounds of salt are in the tank at time t?

4. Consider the Initial Value problem

   (IVP) $$\frac{dy}{dx} = \frac{-x + \sqrt{x^2 + 4y}}{2}, \quad y(2) = -1.$$

   (a) Is $y_1(x) = 1 - x$ a solution of (IVP)? Show your work. Your answer must make sense.
   (b) Is $y_2(x) = \frac{-x^2}{4}$ a solution of (IVP)? Show your work. Your answer must make sense.
   (c) State the Existence and Uniqueness Theorem for first order differential equations.
   (d) Comment on your answer to (c) in light of your answers to (a) and (b).

5. Newton's law of cooling states that the rate at which an object cools is proportional to the difference in temperature between the object and the surrounding medium. Thus, if an object is taken from an oven at $350°$ F and left to cool in a room at $65°$ F, then its temperature T after t hours will satisfy the differential equation

   $$\frac{dT}{dt} = k(T - 65).$$

   If the temperature fell to $250°$ F after one hour, what will it be after 3 hours?