Write everything on the blank paper that you brought. There should be nothing on your desk except this exam, the blank paper that you brought, and a pen or pencil. When you are finished, send a picture of your solutions to

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The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please  $\boxed{CIRCLE}$  your answer. Please **CHECK** your answer whenever possible.

(1) Solve the Initial Value Problem:

$$\frac{dy}{dx} = \frac{x}{(x^2+1)^2}$$
 and  $y(0) = 1$ .

## Please check your answer.

(2) Solve the Initial Value Problem:

$$\frac{dy}{dx} = \frac{2xy}{x^2 + 1} \qquad \text{and} \qquad y(0) = 3.$$

## Please check your answer.

(3) Consider the Initial Value Problem:

$$\frac{dy}{dx} = \frac{x}{y}$$
 and  $y(1) = 2$ 

Use Euler's method to approximate  $y(\frac{3}{2})$ . Use two steps. Make each step size be 1/4.

- (4) Newton's law of cooling states that the rate at which an object cools is proportional to the difference between the temperature of the object and the temperature of the surrounding medium. When a cake is removed from an oven, its temperature is measured at 300 degrees. Three minutes later its temperature is 200 degrees. How long will it take for the cake to cool off to a temperature of 80 degrees? The room temperature is 70 degrees.
- (5) A 1500 gallon tank initially contains 600 gallons of water with 5 pounds of salt dissolved in it. A salt water solution which contains 15 pounds of salt per gallon enters the tank at a rate of 9 gallons per hour. If a well mixed solution leaves the tank at a rate of 6 gallons per hour, how much salt is in the tank when the tank overflows?