Math 242, Exam 1, Fall 2012

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones.

The solutions will be posted later today.

1. (7 points) Find all constants r for which $y = e^{rx}$ is a solution of $3y'' + 3y' - 4y = 0$.

2. (7 points) On the planet Gzyx, a ball dropped from a height of 40 ft hits the ground in 3 seconds. If a ball is dropped from the top of a 200-ft-tall building on Gzyx, how long will it take to hit the ground? With what speed will it hit? **I expect you to solve initial value problems. Unexplained, random formulas will not be accepted!** (Recall that Newton's Law of Motion states that if $F(t)$ is the force acting on an object moving in a straight line at time t, m is the mass of the object, and $a(t)$ is the acceleration of the object at time t, then $F = ma$. The only force acting on this ball on planet Gzyx is the force of gravity and this force is constant.)

3. (7 points) When the brakes are applied to a certain car, the acceleration of the car is $-k\frac{\text{m}}{\text{s}^2}$ for some positive constant k. Suppose that the car is traveling at the velocity $v_0\frac{\text{m}}{\text{s}}$ when the brakes are first applied and that the brakes continue to be applied until the car stops. **I expect you to solve initial value problems. Unexplained, random formulas will not be accepted!**
   (a) Find the distance that the car travels between the moment that the brakes are first applied and the moment when the car stops. (Your answer will be expressed in terms of k and v_0.)
   (b) How does the stopping distance change if the initial velocity is changed to $5v_0$?

Please turn over.

4. (8 points)
   (a) State the Existence and Uniqueness Theorem for first order differential equations.
   (b) What does the Existence and Uniqueness Theorem tell you about the Initial Value Problem
   $$\frac{dy}{dx} = 4x^3y - y \quad y(1) = -3?$$
   (c) Solve the Initial Value Problem of part (b).
   (d) What does the Existence and Uniqueness Theorem tell you about the Initial Value Problem
   $$\frac{dy}{dx} = 4x^3y - y \quad y(1) = 0?$$
   (e) Solve the Initial Value Problem of part (d).

5. (7 points) A pitcher of buttermilk initially at 35° C is to be cooled by setting it on the front porch, where the temperature is 5° C. Suppose that the temperature of the buttermilk has dropped to 25° C after 20 minutes. When will the temperature of the buttermilk reach 10° C? **I expect you to solve initial value problems. Unexplained, random formulas will not be accepted!** (Recall that Newton's Law of Cooling states that the rate at which an object cools is proportional to the difference between the temperature of the object and the temperature of the surrounding medium.)

6. (7 points) Find the general solution of $(x^2+1)\frac{dy}{dx} + 3xy = 6x$. Express your answer in the form $y = y(x)$. **Check your answer.**

7. (7 points) A 120-gallon (gal) tank initially contains 90 pounds (lb) of salt dissolved in 90 gal of water. Brine containing 2 lb/gal of salt flows into the tank at the rate of 4 gal/min, and the well-stirred mixture flows out of the tank at the rate of 3 gal/min. How much salt does the tank contain when the tank becomes full?