5. Compute \[ \int_{1}^{3} \int_{-y}^{2y} x e^{y^3} \, dx \, dy. \]

We see that

\[
\int_{1}^{3} \int_{-y}^{2y} x e^{y^3} \, dx \, dy = \int_{1}^{3} \frac{x^2}{2} e^{y^3} \Bigg|_{-y}^{2y} \, dy = \frac{1}{2} \int_{1}^{3} 3y^2 e^{y^3} \, dy = \frac{1}{2} e^{y^3} \Bigg|_{1}^{3} \, dy = \frac{1}{2} (e^{27} - e).
\]

6. Compute \[ \int_{0}^{4} \int_{\sqrt{3}}^{2} \sin(y^3) \, dy \, dx. \]

We can not express \( \int \sin(y^3) \, dy \) in terms of elementary functions. The present integral uses vertical lines to fill up the region. We switch and use horizontal lines. See the picture on a separate page. The integral is equal to

\[
\int_{0}^{2} \int_{0}^{y^2} \sin(y^3) \, dx \, dy = \int_{0}^{2} x \sin(y^3) \Bigg|_{0}^{y^2} \, dy = \int_{0}^{2} y^2 \sin(y^3) \, dy = - \frac{1}{3} \cos(y^3) \Bigg|_{0}^{2} = - \frac{1}{3} (\cos 8 - 1) = \frac{1}{3} (1 - \cos 8).
\]