Math 241  Exam 2  Spring 2008

TAKE THESE QUESTIONS HOME WITH YOU WHEN YOU LEAVE. I WILL POST SOLUTIONS LATER TODAY.
Write your answers as legibly as you can on the blank sheets of paper provided. Use only one side of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

There are 7 problems. Most of the problems are worth 7 points. The exam is worth 50 points. SHOW your work. Make your work be coherent and clear. Write in complete sentences whenever this is possible. CIRCLE your answer. CHECK your answer whenever possible. No Calculators.

1. Find the directional derivative of \( f(x, y) = x \ln(x + y) \) at the point \((1, 2)\) in the direction of the vector \( \mathbf{a} = 2 \mathbf{i} + 3 \mathbf{j} \).

2. Find the equation of the plane tangent to \( z^2 = x^2 + y^2 \) at the point \((3, 4, 5)\).

3. Find all points of intersection of the line \( x = -1 + t, \ y = 2 + t, \ z = 2t + 7 \) and the surface \( z = x^2 + y^2 \). Please check your answer.

4. Find the equation of the plane that contains the lines \( x = -2 + t, \ y = 3 + 2t, \ z = 4 - t, \) and \( x = 3 - t, \ y = 4 - 2t, \ z = t \). Please check your answer.

5. (8 points) The temperature of a plate at the point \((x, y)\) is \( T(x, y) = 100 + x^2 - y^2 \). Find the path that a heat seeking particle would travel if it starts at the point \((5, \sqrt{75})\). (The particle always moves in the direction of the greatest increase in temperature.)

6. (a) Find \( \lim_{(x,y) \to (0,0)} \frac{x^3 y}{x^6 + 2y^2} \) along \( y = 3x \).

(b) Find \( \lim_{(x,y) \to (0,0)} \frac{x^3 y}{x^6 + 2y^2} \) along \( y = x^3 \).

7. Let \( \mathbf{a} = \mathbf{i} + 2 \mathbf{j} + 3 \mathbf{k} \) and \( \mathbf{b} = 3 \mathbf{i} + 7 \mathbf{j} + 13 \mathbf{k} \). Find vectors \( \mathbf{u} \) and \( \mathbf{v} \) with \( \mathbf{b} = \mathbf{u} + \mathbf{v} \), \( \mathbf{u} \) parallel to \( \mathbf{a} \), and \( \mathbf{v} \) perpendicular to \( \mathbf{a} \). Please check your answer.